

Algebraic Number Theory

(PARI-GP version 2.19.0)

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ **Qfb**(a, b, c) or **Qfb**($[a, b, c]$)
reduce x ($s = \sqrt{D}$, $l = \lfloor s \rfloor$) **qfbred**($x, \{flag\}, \{D\}, \{l\}, \{s\}$)
return $[y, g]$, $g \in \text{SL}_2(\mathbf{Z})$, $y = g \cdot x$ reduced **qfbreds12**(x)
composition of forms $x*y$ or **qfbnucomp**(x, y, l)
 n -th power of form x^n or **qfbnupow**(x, n)
composition **qfbcomp**(x, y)
... without reduction **qfbcomppraw**(x, y)
 n -th power **qfbpow**(x, n)
... without reduction **qfbpowraw**(x, n)
prime form of disc. x above prime p **qfbprimeform**(x, p)
class number of disc. x **qfbclassno**(x)
Hurwitz class number of disc. x **qfbhclassno**(x)
solve $Q(x, y) = n$ in integers **qfbsolve**(Q, n)
solve $x^2 + Dy^2 = p$, p prime **qfbcornacchia**(D, p)
... $x^2 + Dy^2 = 4p$, p prime **qfbcornacchia**($D, 4 * p$)

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$ **quadgen**(x)
minimal polynomial of ω **quadpoly**(x)
discriminant of $\mathbf{Q}(\sqrt{x})$ **quaddisc**(x)
regulator of real quadratic field **quadregulator**(x)
fundamental unit in O_D , $D > 0$ **quadunit**($D, \{w\}$)
norm of fundamental unit in O_D **quadunitnorm**(D)
index of $O_{Df^2}^\times$ in $O_D^\times$ **quadunitindex**(D, f)
class group of $\mathbf{Q}(\sqrt{D})$ **quadclassunit**($D, \{flag\}, \{t\}$)
Hilbert class field of $\mathbf{Q}(\sqrt{D})$ **quadhilbert**($D, \{flag\}$)
... using specific class invariant ($D < 0$) **polclass**($D, \{inv\}$)
test if T is **polclass**(D); if so return D **polisclass**(T)
ray class field modulo f of $\mathbf{Q}(\sqrt{D})$ **quadray**($D, f, \{flag\}$)

General Number Fields: Initializations

The number field $K = \mathbf{Q}[X]/(f)$ is given by irreducible $f \in \mathbf{Q}[X]$.
We denote $\theta = \bar{X}$ the canonical root of f in K . A nf structure
contains a maximal order and allows operations on elements and
ideals. A bnf adds class group and units. A bnr is attached to ray
class groups and class field theory. A rmf is attached to relative
extensions L/K .

init number field structure nf **nfinit**($f, \{flag\}$)
known integer basis B **nfinit**($[f, B]$)
order maximal at $vp = [p_1, \dots, p_k]$ **nfinit**($[f, vp]$)
order maximal at all $p \leq P$ **nfinit**($[f, P]$)
certify maximal order **nfcertify**(nf)

nf members:

a monic $F \in \mathbf{Z}[X]$ defining K **nf.pol**
number of real/complex places **nf.r1/r2/sign**
discriminant of nf **nf.disc**
primes ramified in nf **nf.p**
 T_2 matrix **nf.t2**
complex roots of F **nf.roots**
integral basis of $\mathbf{Z}_K$ as powers of θ **nf.zk**
different/codifferent **nf.diff**, **nf.codiff**
index $[\mathbf{Z}_K : \mathbf{Z}[X]/(F)]$ **nf.index**
recompute nf using current precision **nfnewprec**(nf)
init relative rmf $L = K[Y]/(g)$ **rnfininit**(nf, g)
init bnf structure **bnfininit**(f, l)

bnf members: same as nf , plus
underlying nf **bnf.nf**
class group, regulator **bnf.clgp**, **bnf.reg**
fundamental/torsion units **bnf.fu**, **bnf.tu**
add S -class group and units, yield $bnfS$ **bnfsunit**(bnf, S)
init class field structure bnr **bnrinit**($bnf, m, \{flag\}$)
bnr members: same as bnf , plus
underlying bnf **bnr.bnf**
big ideal structure **bnr.bid**
modulus m **bnr.mod**
structure of $(\mathbf{Z}_K/m)^*$ **bnr.zkst**

Fields, subfields, embeddings

Defining polynomials, embeddings
(some) number fields with Galois group G **nflist**(G)
... and $|\text{disc}(K)| = N$ and s complex places **nflist**($G, N, \{s\}$)
... and $a \leq |\text{disc}(K)| \leq b$ **nflist**($G, [a, b], \{s\}$)
smallest poly defining $f = 0$ (slow) **polredabs**($f, \{flag\}$)
small poly defining $f = 0$ (fast) **polredbest**($f, \{flag\}$)
monic integral $g = Cf(x/L)$ **polmonic**($f, \{\&L\}$)
random Tschirnhausen transform of f **poltschirnhaus**(f)
 $\mathbf{Q}[t]/(f) \subset \mathbf{Q}[t]/(g)$? Isomorphic? **nfisincl**(f, g), **nfisisom**
reverse polmod $a = A(t) \bmod T(t)$ **modreverse**(a)
compositum of $\mathbf{Q}[t]/(f)$, $\mathbf{Q}[t]/(g)$ **polcompositum**($f, g, \{flag\}$)
compositum of $K[t]/(f)$, $K[t]/(g)$ **nfcompositum**($nf, f, g, \{flag\}$)
splitting field of K (degree divides d) **nfsplitting**($nf, \{d\}$)
signs of real embeddings of x **nfeltsign**($nf, x, \{pl\}$)
complex embeddings of x **nfeltembed**($nf, x, \{pl\}$)
 $T \in K[t]$, # of real roots of $\sigma(T) \in R[t]$ **nfpolsturm**($nf, T, \{pl\}$)
absolute Weil height **nfweilheight**(nf, v)

Subfields, polynomial factorization

subfields (of degree d) of nf **nfsubfields**($nf, \{d\}$)
maximal subfields of nf **nfsubfieldsmax**(nf)
maximal CM subfield of nf **nfsubfieldscm**(nf)
 $K_d \subset \mathbf{Q}(\zeta_n)$, using Gaussian periods **polsubcyclo**($n, d, \{v\}$)
... using class field theory **polsubcyclofast**(n, d)
roots of unity in nf **nfroots01**(nf)
roots of g belonging to nf **nfroots**(nf, g)
factor g in nf **nfactor**(nf, g)

Linear and algebraic relations

poly of degree $\leq k$ with root $x \in \mathbf{C}$ or $\mathbf{Q}_p$ **algdep**(x, k)
alg. dep. with pol. coeffs for series s **seralgdep**(s, x, y)
diff. dep. with pol. coeffs for series s **serdiffdep**(s, x, y)
small linear rel. on coords of vector x **lindep**(x)

Basic Number Field Arithmetic (nf)

Number field elements are **t_INT**, **t_FRAC**, **t_POL**, **t_POLMOD**, or **t_COL**
(on integral basis $nf.zk$).

Basic operations

$x + y$ **nfeltadd**(nf, x, y)
 $x \times y$ **nfeltmul**(nf, x, y)
 x^n , $n \in \mathbf{Z}$ **nfeltpow**(nf, x, n)
 x/y **nfeltdiv**(nf, x, y)
 $q = x \backslash y := \text{round}(x/y)$ **nfeltdivouc**(nf, x, y)
 $r = x \% y := x - (x \backslash y)y$ **nfeltmod**(nf, x, y)
... $[q, r]$ as above **nfeltdivrem**(nf, x, y)
reduce x modulo ideal A **nfeltreduce**(nf, x, A)
absolute trace $\text{Tr}_{K/\mathbf{Q}}(x)$ **nfelttrace**(nf, x)
absolute norm $N_{K/\mathbf{Q}}(x)$ **nfeltnorm**(nf, x)

is x a square? **nfeltissquare**($nf, x, \{\&y\}$)
... an n -th power? **nfeltispower**($nf, x, n, \{\&y\}$)

Multiplicative structure of K^* ; $K^*/(K^*)^n$

valuation $v_{\mathfrak{p}}(x)$ **nfeltval**($nf, x, \mathfrak{p}$)
... write $x = \pi^{v_{\mathfrak{p}}(x)}y$ **nfeltval**($nf, x, \mathfrak{p}, \&y$)
quadratic Hilbert symbol (at $\mathfrak{p}$) **nfhilbert**($nf, a, b, \{\mathfrak{p}\}$)
 b such that $xb^n = v$ is small **idealredmodpower**(nf, x, n)

Maximal order and discriminant

integral basis of field $\mathbf{Q}[x]/(f)$ **nfbasis**(f)
field discriminant of $\mathbf{Q}[x]/(f)$ **nfdisc**(f)
... and factorization **nfdiscfactors**(f)
express x on integer basis **nfalgtobasis**(nf, x)
express element x as a polmod **nfbasistoalg**(nf, x)

Hecke Grossencharacters

Let K be a number field and m a modulus. A **gchar** structure
describes the group of Hecke Grossencharacters of K of modulus m
and allows computations with these characters. A character χ is
described by its components modulo $gc.cyc$.

init **gchar** structure gc for modulus m **gcharinit**($bnf, m, \{cm\}$)

gc members:

underlying bnf **gc.bnf**
modulus **gc.mod**
elementary divisors (including 0s) **gc.cyc**
recompute gc using current precision **gcharnewprec**(gc)
evaluate Hecke character chi at ideal id **gchareval**(gc, chi, id)
exponent column of id in $\mathbf{R}^n$ **gcharideallog**(gc, id)
log representation of ideal id **gcharlog**(gc, id)
... of character χ **gcharduallog**(gc, chi)
exponent vector of χ in $\mathbf{R}^n$ **gcharparameters**(gc, chi)
conductor of χ **gcharconductor**(gc, chi)
L-function of χ **lfuncreate**($[gc, chi]$)
local component χ_v of χ **gcharlocal**(gc, chi, v)
 χ s.t. $\chi_v \approx Lchiv[i]$ for $v = Lv[i]$ **gcharidentify**($gc, Lv, Lchiv$)
basis of group of algebraic characters **gcharalgebraic**(gc)
algebraic character of given infinity type **gcharalgebraic**($gc, type$)
is χ algebraic? **gcharisalgebraic**(gc, chi)

Dedekind Zeta Function ζ_K , Hecke L series

$R = [c, w, h]$ in initialization means we restrict $s \in \mathbf{C}$ to domain
 $|\Re(s) - c| < w$, $|\Im(s)| < h$; $R = [w, h]$ encodes $[1/2, w, h]$ and $[h]$
encodes $R = [1/2, 0, h]$ (critical line up to height h).

ζ_K as Dirichlet series, $N(I) \leq b$ **dirzetak**(nf, b)
init $\zeta_K^{(k)}(s)$ for $k \leq n$ **L = lfunitinit**($bnf, R, \{n = 0\}$)
compute $\zeta_K(s)$ (n -th derivative) **lfun**($L, s, \{n = 0\}$)
compute $\Lambda_K(s)$ (n -th derivative) **lfunlambda**($L, s, \{n = 0\}$)

init $L_K^{(k)}(s, \chi)$ for $k \leq n$ **L = lfunitinit**($[bnr, chi], R, \{n = 0\}$)
compute $L_K(s, \chi)$ (n -th derivative) **lfun**($L, s, \{n\}$)
Artin root number of K **bnrrootnumber**($bnr, chi, \{flag\}$)
 $L(1, \chi)$, for all χ trivial on H **bnrL1**($bnr, \{H\}, \{flag\}$)

Class Groups & Units (bnf, bnr)

Class field theory data $a_1, \{a_2\}$ is usually bnr (ray class field),
 bnr, H (congruence subgroup) or bnr, χ (character on **bnr.clgp**).
Any of these define a unique abelian extension of K .
units / S -units **bnfunits**($bnf, \{S\}$)
remove GRH assumption from bnf **bnfcertify**(bnf)

expo. of ideal x on class gp `bnfisprincipal(bnf,x,{flag})`
...on ray class gp `bnrisprincipal(bnr,x,{flag})`
expo. of x on fund. units `bnfisunit(bnf,x)`
...on S -units, U is `bnfunits(bnf,S)` `bnfisunit(bnfs,x,U)`
signs of real embeddings of bnf .fu `bnfsignunit(bnf)`
narrow class group `bnfnarrow(bnf)`

Class Field Theory

ray class number for modulus m `bnrclassno(bnf,m)`
discriminant of class field `bnrdisc(a1,{a2})`
ray class numbers, l list of moduli `bnrclassnolist(bnf,l)`
discriminants of class fields `bnrdisclist(bnf,l,{arch},{flag})`
decode output from `bnrdisclist` `bnfdecodemodule(nf,fa)`
is modulus the conductor? `bnrisconductor(a1,{a2})`
is class field (bnr,H) Galois over K^G `bnrisgalois(bnr,G,H)`
action of automorphism on `bnr.gen` `bnrgaloismatrix(bnr,aut)`
apply `bnrgaloismatrix M` to H `bnrgaloisapply(bnr,M,H)`
characters on `bnr.clgp` s.t. $\chi(g_i) = e(v_i)$ `bnrchar(bnr,g,{v})`
conductor of character χ `bnrconductor(bnr,chi)`
conductor of extension `bnrconductor(a1,{a2},{flag})`
conductor of extension $K[Y]/(g)$ `rnfconductor(bnf,g)`
canonical projection $Cl_F \rightarrow Cl_f, f \mid F$ `bnrmap`
Artin group of extension $K[Y]/(g)$ `rnfnormgroup(bnr,g)`
subgroups of bnr , index $\leq b$ `subgrouplist(bnr,b,{flag})`
compositum as `[bnr,H]` `bnrcompositum([bnr1,H1],[bnr2,H2])`
class field defined by $H \subset Cl_f$ `bnrclassfield(bnr,H)`
...low level equivalent, prime degree `rnfkummer(bnr,H)`
same, using Stark units (real field) `bnrstark(bnr,{sub},{flag})`
Stark unit `bnrstarkunit(bnr,{sub})`
is a an n -th power in K_v ? `nfislocalpower(nf,v,a,n)`
cyclic L/K satisf. local conditions `nfgrunwaldwang(nf,P,D,pl)`

Cyclotomic and Abelian fields theory

An Abelian field F given by a subgroup $H \subset (Z/fZ)^*$ is described by an argument F , e.g. f (for $H = 1$, i.e. $Q(\zeta_f)$) or $[G,H]$, where G is `idealstar(f,1)`, or a minimal polynomial.

minus class number $h^-(F)$ `subcyclohminus(F)`
... p -part `subcyclohminus(F,p)`
minus part of Iwasawa polynomials `subcycloiwasa(F,p)`
 p -Sylow of $Cl(F)$ `subcyclopclgp(F,p)`
is p coprime to $h(Q(\zeta_p))$? `primeisregular(p)`

Logarithmic class group

logarithmic ℓ -class group `bnflog(bnf,l)`
 $[\tilde{e}(F_v/Q_p), \tilde{f}(F_v/Q_p)]$ `bnflogef(bnf,pr)`
 $\exp \deg_F(A)$ `bnflogdegree(bnf,A,l)`
is ℓ -extension L/K locally cyclotomic `rnfislocalcyclo(rnf)`

Ideals: elements, primes, or matrix of generators in HNF

is id an ideal in nf ? `nfisideal(nf,id)`
is x principal in bnf ? `bnfisprincipal(bnf,x)`
give $[a,b]$, s.t. $aZ_K + bZ_K = x$ `idealtwoelt(nf,x,{a})`
put ideal a ($aZ_K + bZ_K$) in HNF `idealhnf(nf,a,{b})`
HNF for ideal generated by $a_1, \dots, a_n$ `idealfromgens(nf,a)`
norm of ideal x `idealnrm(nf,x)`
minimum of ideal x (direction v) `idealmin(nf,x,v)`
LLL-reduce the ideal x (direction v) `idealred(nf,x,{v})`

Algebraic Number Theory

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Ideal Operations

add ideals x and y `idealadd(nf,x,y)`
multiply ideals x and y `idealmul(nf,x,y,{flag})`
intersection of ideal x with Q `idealdown(nf,x)`
intersection of ideals x and y `idealintersect(nf,x,y,{flag})`
 n -th power of ideal x `idealpow(nf,x,n,{flag})`
inverse of ideal x `idealinv(nf,x)`
divide ideal x by y `idealdiv(nf,x,y,{flag})`
Find $(a,b) \in x \times y, a + b = 1$ `idealaddtoone(nf,x,{y})`
coprime integral A,B such that $x = A/B$ `idealnumden(nf,x)`

Primes and Multiplicative Structure

check whether x is a maximal ideal `idealismaximal(nf,x)`
factor ideal x in Z_K `idealfactor(nf,x)`
expand ideal factorization in K `idealfactorback(nf,f,{e})`
is ideal A an n -th power ? `idealispower(nf,A,n)`
expand elt factorization in K `nfactorback(nf,f,{e})`
decomposition of prime p in Z_K `idealprimedec(nf,p)`
valuation of x at prime ideal pr `idealval(nf,x,pr)`
weak approximation theorem in nf `idealchinese(nf,x,y)`
 $a \in K$, s.t. $v_p(a) = v_p(x)$ if $v_p(x) \neq 0$ `idealappr(nf,x)`
 $a \in K$ such that $(a \cdot x, y) = 1$ `idealcoprime(nf,x,y)`
give bid =structure of $(Z_K/id)^*$ `idealstar(nf,id,{flag})`
structure of $(1 + \mathfrak{p})/(1 + \mathfrak{p}^k)$ `idealprincipalunits(nf,pr,k)`
discrete log of x in $(Z_K/bid)^*$ `ideallog(nf,x,bid)`
`idealstar` of all ideals of norm $\leq b$ `ideallist(nf,b,{flag})`
add Archimedean places `ideallistarch(nf,b,{ar},{flag})`
init `modpr` structure `nfmodprinit(nf,pr,{v})`
project t to Z_K/pr `nfmodpr(nf,t,modpr)`
lift from Z_K/pr `nfmodprlift(nf,t,modpr)`

Galois theory over Q

conjugates of a root θ of nf `nfgaloisconj(nf,{flag})`
apply Galois automorphism s to x `nfgaloisapply(nf,s,x)`
Galois group of field $Q[x]/(f)$ `polgalois(f)`
resolvent field of $Q[x]/(f)$ `nfresolvent(f)`
initializes a Galois group structure G `galoisinit(pol,{den})`
...for the splitting field of pol `galoissplittinginit(pol,{d})`
character table of G `galoischartable(G)`
conjugacy classes of G `galoisconjclasses(G)`
 $\det(1 - \rho(g)T)$, χ character of ρ `galoischarpoly(G,\chi,{o})`
 $\det(\rho(g))$, χ character of ρ `galoischardet(G,\chi,{o})`
action of p in `nfgaloisconj` form `galoispermtpol(G,{p})`
identify as abstract group `galoisidentify(G)`
export a group for GAP/MAGMA `galoisexport(G,{flag})`
subgroups of the Galois group G `galoissubgroups(G)`
order of a group or subgroup `galoisorder(H)`
is subgroup H normal? `galoisisnormal(G,H)`
subfields from subgroups `galoissubfields(G,{flag},{v})`
fixed field `galoisfixedfield(G,perm,{flag},{v})`
Frobenius at maximal ideal P `idealfrobenius(nf,G,P)`
ramification groups at P `idealramgroups(nf,G,P)`
is G abelian? `galoisisabelian(G,{flag})`
abelian number fields/ Q `galoissubcyclo(N,H,{flag},{v})`

The galpol package

query the package: polynomial `galoisgetpol(a,b,{s})`
...: permutation group `galoisgetgroup(a,b)`
...: group description `galoisgetname(a,b)`

Relative Number Fields (rnf)

Extension L/K is defined by $T \in K[x]$.
absolute equation of L `rnfequation(nf,T,{flag})`
is L/K abelian? `rnfisabelian(nf,T)`
relative `nfalgtobasis` `rnfalgtobasis(rnf,x)`
relative `nfbasistoalg` `rnfbasistoalg(rnf,x)`
relative `idealhnf` `rnfidealhnf(rnf,x)`
relative `idealmul` `rnfidealmul(rnf,x,y)`
relative `idealtwoelt` `rnfidealtwoelt(rnf,x)`

Lifts and Push-downs

absolute $\rightarrow$ relative representation for x `rnfeltabstorel(rnf,x)`
relative $\rightarrow$ absolute representation for x `rnfeltretloabs(rnf,x)`
lift x to the relative field `rnfeltup(rnf,x)`
push x down to the base field `rnfeltdown(rnf,x)`
idem for x ideal: `(rnfideal)reltoabs, abstorel, up, down`

Norms and Trace

relative norm of element $x \in L$ `rnfeltnorm(rnf,x)`
relative trace of element $x \in L$ `rnfelttrace(rnf,x)`
absolute norm of ideal x `rnfidealnrmabs(rnf,x)`
relative norm of ideal x `rnfidealnrmrel(rnf,x)`
solutions of $N_{K/Q}(y) = x \in Z$ `bnfisintnorm(bnf,x)`
is $x \in Q$ a norm from K ? `bnfisnorm(bnf,x,{flag})`
initialize T for norm eq. solver `rnfisnorminit(K,pol,{flag})`
is $a \in K$ a norm from L ? `rnfisnorm(T,a,{flag})`
initialize t for Thue equation solver `thueinit(f)`
solve Thue equation $f(x,y) = a$ `thue(t,a,{sol})`
characteristic poly. of $a \bmod T$ `rnfcharpoly(nf,T,a,{v})`

Factorization

factor ideal x in L `rnfidealfactor(rnf,x)`
 $[S,T]:T_{i,j} \mid S_i; S$ primes of K above p `rnfidealprimedec(rnf,p)`

Maximal order Z_L as a Z_K -module

relative `polredbest` `rnfpolredbest(nf,T)`
relative `polredabs` `rnfpolredabs(nf,T)`
relative Dedekind criterion, prime pr `rnfdedekind(nf,T,pr)`
discriminant of relative extension `rnfdisc(nf,T)`
pseudo-basis of Z_L `rnfpseudobasis(nf,T)`

General Z_K -modules: $M = [\text{matrix, vec. of ideals}] \subset L$
relative HNF / SNF `nfhnf(nf,M), nfsnf`
multiple of $\det M$ `nfdetint(nf,M)`
HNF of M where $d = nfdetint(M)$ `nfhnfmod(x,d)`
reduced basis for M `rnfilllgram(nf,T,M)`
determinant of pseudo-matrix M `rnfdet(nf,M)`
Steinitz class of M `rnfsteinitz(nf,M)`
 Z_K -basis of M if Z_K -free, or 0 `rnfhnfbasis(bnf,M)`
 n -basis of M , or $(n + 1)$ -generating set `rnfbasis(bnf,M)`
is M a free Z_K -module? `rnfisfree(bnf,M)`

Associative Algebras

A is a general associative algebra given by a multiplication table mt (over $\mathbf{Q}$ or $\mathbf{F}_p$); represented by al from `algtableinit`.

create al from mt (over $\mathbf{F}_p$) `algtableinit(mt, {p = 0})`
group algebra $\mathbf{Q}[G]$ (or $\mathbf{F}_p[G]$) `alggroup(G, {p = 0})`
center of group algebra `alggrouppcenter(G, {p = 0})`

Properties

is (mt, p) OK for `algtableinit`? `algisassociative(mt, {p = 0})`
multiplication table mt `algmultable(al)`
dimension of A over prime subfield `algdim(al)`
characteristic of A `algchar(al)`
is A commutative? `algiscommutative(al)`
is A simple? `algissimple(al)`
is A semi-simple? `algissemisimple(al)`
center of A `algcenter(al)`
Jacobson radical of A `algradical(al)`
radical J and simple factors of A/J `algsimpledec(al)`

Operations on algebras

create A/I , I two-sided ideal `algquotient(al, I)`
create $A_1 \otimes A_2$ `algtensor(al1, al2)`
create subalgebra from basis B `algsubalg(al, B)`
quotients by ortho. central idempotents e `algcentralproj(al, e)`
isomorphic alg. with integral mult. table `algmakeintegral(mt)`
prime subalgebra of semi-simple A over $\mathbf{F}_p$ `algprimesubalg(al)`
find isomorphism $A \cong M_d(\mathbf{F}_q)$ `algsplit(al)`

Operations on lattices in algebras

lattice generated by cols. of M `alglathnf(al, M)`
... by the products xy , $x \in lat1$, $y \in lat2$ `alglatmul(al, lat1, lat2)`
sum $lat1 + lat2$ of the lattices `alglatadd(al, lat1, lat2)`
intersection $lat1 \cap lat2$ `alglatinter(al, lat1, lat2)`
test $lat1 \subset lat2$ `alglatsubset(al, lat1, lat2)`
generalized index $(lat2 : lat1)$ `alglatindex(al, lat1, lat2)`
 $\{x \in al \mid x \cdot lat1 \subset lat2\}$ `alglatlefttransporter(al, lat1, lat2)`
 $\{x \in al \mid lat1 \cdot x \subset lat2\}$ `alglatrighttransporter(al, lat1, lat2)`
test $x \in lat$ (set $c = \text{coord. of } x$) `alglatcontains(al, lat, x, {\&c})`
element of lat with coordinates c `alglatelement(al, lat, c)`

Operations on elements

$a + b$, $a - b$, $-a$ `algadd(al, a, b)`, `algsub`, `algneg`
 $a \times b$, a^2 `algmul(al, a, b)`, `algsqr`
 a^n , a^{-1} `algpow(al, a, n)`, `alginv`
is x invertible ? (then set $z = x^{-1}$) `alginv(al, x, {\&z})`
find z such that $x \times z = y$ `algdivl(al, x, y)`
find z such that $z \times x = y$ `algdivr(al, x, y)`
does z s.t. $x \times z = y$ exist? (set it) `algdivl(al, x, y, {\&z})`
matrix of $v \mapsto x \cdot v$ `algtomatrix(al, x)`
absolute norm `algnorm(al, x)`
absolute trace `algtrace(al, x)`
absolute char. polynomial `algcharpoly(al, x)`
given $a \in A$ and polynomial T , return $T(a)$ `algpoleval(al, T, a)`
random element in a box `algrandom(al, b)`

Central Simple Algebras

A is a central simple algebra over a number field K ; represented by al from `alginitt`; K is given by a nf structure.

create CSA from data `alginitt(B, C, {v}, {flag = 3})`
multiplication table over K $B = K$, $C = mt$
cyclic algebra $(L/K, \sigma, b)$ $B = rnf$, $C = [sigma, b]$
quaternion algebra $(a, b)_K$ $B = K$, $C = [a, b]$
matrix algebra $M_d(K)$ $B = K$, $C = d$
local Hasse invariants over K $B = K$, $C = [d, [PR, HF], HI]$
ramification set of quaternion algebra $B = K$, $C = [PR, HI]$
Hamilton quaternion algebra $(-1, -1)_{\mathbf{R}}$ $B = 1.$, $C = 1/2$
recover (a, b) if $al = (a, b)_K$ `algisquatalg(al)`
recompute al using current precision `algnewprec(al)`

Properties

type of al (mt , CSA) `algtype(al)`
dimension of A over $\mathbf{Q}$ `algdim(al, 1)`
dimension of al over its center K `algdim(al)`
degree of A ($= \sqrt{\dim_K A}$) `algdegree(al)`
 al a cyclic algebra $(L/K, \sigma, b)$; return σ `algaut(al)`
... return b `algb(al)`
... return L/K , as an rnf `algsplittingfield(al)`
split A over an extension of K `algsplittingdata(al)`
splitting field of A as an rnf over center `algsplittingfield(al)`
multiplication table over center `algrelmultable(al)`
places of K at which A ramifies `algramifiedplaces(al)`
Hasse invariants at finite places of K `alghassef(al)`
Hasse invariants at infinite places of K `alghassei(al)`
Hasse invariant at place v `alghasse(al, v)`
index of A over K (at place v) `algindex(al, {v})`
is al a division algebra? (at place v) `algisdivision(al, {v})`
is A ramified? (at place v) `algisramified(al, {v})`
is A split? (at place v) `algissplit(al, {v})`
is $A \cong A_2$? (at place v) `algisisom(al, al2, {v})`

Operations on elements

$x \in nf$ as an element of al `algeltfromnf(al, x)`
reduced norm `algnorm(al, x)`
reduced trace `algtrace(al, x)`
reduced char. polynomial `algcharpoly(al, x)`
express x on integral basis `algalgtobasis(al, x)`
convert x to algebraic form `algbasistoalg(al, x)`
express quaternion x on integral basis `algquattobasis(al, x)`
convert x to quaternion form `algbasistoquat(al, x)`
quaternion conjugate of x `alginvol(al, x)`
map $x \in A$ to $M_d(L)$, L split. field `algtomatrix(al, x)`
init mod pr map $\mathcal{O}_0 \rightarrow M_k(\mathbf{F}_q)$ `algmodprinit(al, pr, {v})`
project x to $M_k(\mathbf{F}_q)$ `algmodpr(al, x, modP)`
lift from $M_k(\mathbf{F}_q)$ to $\mathcal{O}_0$ `algmodprlift(al, x, modP)`
 $g \in A^\times$ s.t. $fa = gag^{-1}$ `algskolemnoether(al, a, fa)`

Orders

Z-basis of order $\mathcal{O}_0$ `algbasis(al)`
discriminant of order $\mathcal{O}_0$ `algdisc(al)`
Z-basis of natural order in terms $\mathcal{O}_0$'s basis `alginvbasis(al)`
Z-basis of Eichler order of level N `algeichlerbasis(al, N)`

Based on an earlier version by Joseph H. Silverman
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